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A COMPARISON OF PARAMETRIC MORTALITY MODELS TO GRADUATE URBAN AND RURAL MORTALITY

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ABSTRACT
This study examine twenty-nine parametric mortality models and assess their suitability for graduating mortality rates of urban and rural areas in Pakistan. Grouped age specific mortality rates of rural and urban populations for the year 2019 are used. The data is collected from the website of National Institute of Population Studies which conduct Maternal Mortality Survey in Pakistan on regular basis. The parametric mortality models were applied to rural and urban mortality data. We used R software to estimate the model's parameters and assess their suitability for urban and rural populations. The suitability of these models was assessed by using 3 different loss functions. Our analyses found that the fourth type of Heligman-Polard's model with loss function 3 provides reliable results for graduating the mortality of rural population while second type of Carriere model with loss function 3 produce best results for graduating the urban mortality of Pakistan. Based on two models, mortality rates of urban and rural population have been graduated over age range 0-85. We suggest the use the graduated mortality rates of urban and rural areas for pricing life insurance products in rural and urban areas respectively. In addition, graduated mortality rates are also suggested for use in calculation of life insurance liabilities.

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INTRODUCTION

The mortality rates of the population provide important information about that population and are helpful in many aspects. For instant, the life insurance companies use the mortality rates in underwriting (Lew, 1948). Similarly, pension schemes use mortality rates in assessing the risk of longevity (Longevity Risk and Private Pensions, 2007) and policy makers at government level use mortality rates in channelizing funds toward particular segment of population (Miladinov, 2020).Currently, life insurance companies in Pakistan use single mortality rate for pricing their products in urban and rural areas as well as different sexes (PSOA, 2013). However, mortality rates in urban and rural mortalities are diverse in general (Garcia, 2020; Veliuliene & Kalediene, 2021). In the case of Pakistan, there is a difference in the mortality rates of urban and rural areas. For instant, urban population in Pakistan experiences lower mortality up to age 49 and below as compared to rural mortality rates. In this connection, the mortality rates are required on regular basis for pricing life insurance products, valuation on the insurance liabilities etc. in this regard, though, major source of the mortality rates in Pakistan maternal mortality surveys which provides the grouped age specific mortality rates whereas, for pricing insurance products and valuation of the liabilities, the life insurance companies require the mortality rates at each age.

There is a need to graduate mortality rates at each age. Several methods are used to graduate mortality rates like Kernel's method, graphical method, the method of oscillatory interpolation, summation method, the spline method, curve fitting or parametric method, change equation method, graduation by reference to a standard table, and linear programming method (Salhi & Thérond, 2018). The purpose of this article is to graduate the mortality rates of urban and rural population of Pakistan by using parametric mortality models to remove fluctuation in mortality rates and provide smooth mortality rates for pricing and valuation of the liabilities of insurance products. Thus, study examines and assess around twenty-nine different parametric mortality models and propose appropriate parametric mortality models for graduating urban and rural mortalities. Earlier parametric mortality models and their suitability for graduating mortality rates of assured lives have been conducted by Pakistan Society of Actuaries wherein they have compared five parametric mortality models on data from the life insurance companies (PSOA, 2013). The study will provide smooth mortality rates of rural and urban areas of Pakistan and will help life insurance companies in differentiating pricing insurance products in rural and urban areas.

LITERATURE REVIEW

There are two broad categories of graduating mortality rates parametric and non-parametric methods. The parametric mortality models are functions of age and represent the relationship between mortality rates and age (Debon, Montes & Sala, 2006). The first parametric mortality model based on age was proposed by DeMovries (Walford, 1884). Other well-known mortality models include the Gompertz model of force of mortality based on two parameters (Gompertz, 1825), Markham's law of force of mortality with three parameters (Makeham, 1866), Operman model of mortality (Hoem, 1983) with three parameters. More complex model includes Thiele's model which consists of three models of force of mortality at diverse age groups (Thiele, 1871), Wittstein and Bumstead model (Wittstein & Bumsted, 1883), Stefensen (Steffensen, 1930), Wilfred Perks (Perks, 1932), inverse-Weibul, Quadratic, Maen, Weibul distribution claims for modeling mortality (Weibull, 1951), likewise, Beard's force of mortality, Gompertz and Makham's model (Beard, 2008), Vaupeletal Gamma-Gompertz model (Vaupel, Manton & Stallard, 1979), Hougrad (Hougaard, 1984), McGilchrist and Aisbett (Salinari & Santis, 2020) were considered as leading models.

Other complex models include models proposed by Heligman and Pollard with eight parameters of which three parameters addresses the child mortality, the other three parameters addresses accidental humps and two parameters addresses the old age mortality (Heligman & Pollard, 1980), Mode and Busby's eight parameters model of force of mortality for three different age groups (Mode & Busby, 1982), Siler's five parameters model (Siler, 1983), Rogers and Planck's double exponential curve, Perk's formula for the old age mortality (Martinelle, 1986), Carriere's general law of the mortality, Weibul's model, Inverse Gompertz, and Inverse Weibul Survival function (Carriere, 1992), Kostaki's nine parameters version of the Heligman Polard's model (Kostaki, 1991) as well as Kannisto's model (Thatcher, Kannisto & Vaupel, 1999). The list of parametric mortality models is given in table 3. In Pakistan context, amny authors have attempted to develop life tables, for instant, Kuroki and Khan made comparison of Life tables with regional model life table and reliability, validity and stability of the Pakistan life table (Krotki & Khan, 1980).

In addition, abridge life table of Pakistan and provinces by sex was developed by using standard method (Aslam, Hashmi & Seltze, 1962), Gompertz Curve was used to develop the Provisional Abridged Life Tables for Urban and Rural Areas in Pakistan to graduate the mortality rates of rural and urban areas in Pakistan (Farooqui & Alam, 1974), Burney, Akhtar and Aslam (1992) used uniform, exponential and Chiang's method to construct the life table of Pakistan. In 1961, Yusuf et al. developed life table for all Pakistan and provinces using Gompertz Curve (Yusuf & Farooqui, 1969). The descriptive studies on the mortality in Pakistan include work of (Sathar & Mallick, 1991) which has analysed changes in mortality in Pakistan over the period 1966-88. A recent study conducted in 2013 on the application of parametric models for development of life table based on assured lives of insurance companies by the Pakistan Society of Actuaries where they used a few selected mortality parametric models to graduate the selected lives of males underwritten during year 2001-2007 (PSOA, 2013). No other study is conducted that graduate general mortality rates of the urban and rural areas of Pakistan using the parametric mortality models.

RESEARCH METHODOLOGY

Mortality data of urban and rural areas of Pakistan are used in this analysis which is collected from National Institute Population Studies (NIPS) Pakistan. Mortality data is collected over Maternal Mortality Survey of Pakistan from time to time and determine mortality rates based on survey conducted. Data contain grouped age specific data with number of deaths, exposures, and mortality rate at each group. Latest available data on deaths and exposures of all-cause mortality split by urban and rural areas collected over Pakistan Maternal Mortality Survey- 2019 is given table 1.

Table 1

Age-Specific Mortality Rates of Urban and Rural Areas

Age group	Urban	Rural	Age group	Urban	Rural
<1	56.34	66.39	40-44	3.64	4.20
1 - 4	1.91	2.87	45-49	6.44	5.89
5 - 9	0.56	0.93	50-54	12.37	8.83
10-14	0.52	0.74	55-59	17.84	14.84
15-19	1.04	1.16	60-64	32.33	24.36
20-24	1.08	1.34	65-69	41.15	30.63
25-29	1.14	1.67	70-74	72.33	61.68
30-34	1.74	1.82	75-79	68.66	61.66
35-39	1.66	2.61	80+	181.92	164.79

Source: Maternal Mortality Survey of Pakistan, NIPS-2019

We apply twent-nine parametric models as given in table 3 to rural and urban mortality rates. All models are compared for their suitability to graduate the mortality rates of urban and rural areas. The parameters of each model were estimated using likelihood estimation method and estimated values of parameters of each model were used to estimate the mortality rates. Loss functions of the models were calculated. We used three types of loss functions as given in table 2 below:

Table 2
Loss Functions

S. No	Los Function
1	LF1 = $-D_x$ $*(log(1-e^{mu})-(E_x-D_x)*mu)$
2	LF2=log$\left(\frac{mu}{ov}\right)^2$
3	LF3= $\frac{(ov-mu)^2}{Ov}$

Where
D_x: Death counts: E_x: Population exposed to risk: mu: Estimated value of deaths
ov: Observed value of deaths.

A model that provides lowest loss function with lower variation across the three loss functions and assumptions of residuals as given below are validated, is chosen for graduating mortality rates:

1. The residuals are normally distributed
2. The residuals are identically and independently distributed

Date Analysis
Table 2 provides comparison of observed mortality rates of urban and rural areas. It is evident that urban mortality rates up to age 49 are lower than mortality rates of rural population, still, mortality rates beyond age 49 in rural areas is lower than the that of the urban areas. The child mortality rates in rural areas are higher than urban areas. Like any other population, mortality pattern of urban and rural areas increases exponentially with age and thus use of parametric models based on age are justified.

Figure 1
Log Age-specific Mortality Rates by Residence and Region

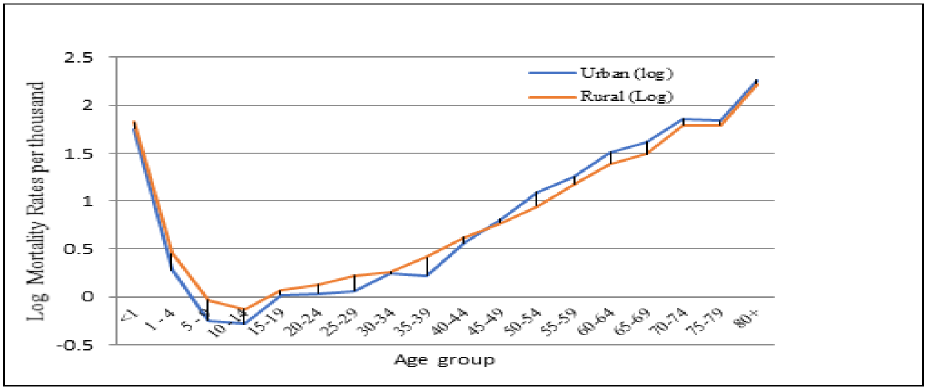


Figure 1 shows log mortality rates of urban and rural areas of Pakistan which indicates that the mortality rates are increasing exponentially with age and thus use of parametric mortality models is justified.

Table 3
Parametric Mortality Models

S. No	YEAR	NAME	Model
1	1825	Gompertz	$\mu_x = Ae^{Bx}$
2	NA	Gompertz	$\mu_x = \frac{1}{sigma} * e^{\frac{(x-M)}{sigma}}$
3	NA	Inv-Gompertz	$\mu_x = \frac{(1 - e^{\frac{(x-M)}{sigma}})}{(e^{\frac{-(x-M)}{sigma}} - 1)}$
4	1860	Makeham	$\mu_x = Ae^{Bx} + C$
5	NA	Makeham	$\mu_x = \frac{1}{sigma} * e^{\frac{(x-M)}{sigma}} + C$
6	1870	Opperman	$\mu_x = \frac{A}{\mathrm{sqrt(x) - B}} + C * sqrt(x)$
7	1871	Thiele	$\mu_x = = Ae^{-Bx} + \mathrm{C}e^{-5D(x-E)^2} + Fe^{Gx}$
8	1883	Wittstein	$q_x = \frac{1}{B} A^{-(Bx^N)} + A^{-(M-X)^N}$
9	1932	Perks	$\mu_x = \frac{A+BC^x}{BC^{-X}+1+DC^x}$
10	1939	Weibull	$\mu_x = \frac{1}{\sigma} \frac{X}{M}^{\frac{M}{\sigma}-1}$
11	NA	Inverse-Weibull	$\mu_x = \frac{\frac{1}{\sigma M} X^{\frac{M}{\sigma}-1}}{e^{\frac{X}{M}^{-\frac{M}{\sigma}-1}}}$
12	1943	Van der Maen	$\mu_x = A + Bx + cx^2 + \frac{I}{N-x}$
13	1943	Van der Maen	$\mu_x = A + Bx + \frac{I}{N-x}$
14	1960	Strehler-Mildvan	$\mu_x = \mathrm{K}e^{-v0\frac{1-Bx}{D}}$
15	NA	Quadratic	$\mu_x = A + Bx + \mathrm{C}x^2$
16	1971	Beard	$\mu_x = \frac{Ae^{Bx}}{1 + KAAe^{Bx}}$
17	1971	Beard-Makeham	$\mu_x = \frac{Ae^{Bx}}{1 + KAAe^{Bx}} + C$
18	1979	Gamma-Gompertz	$\mu_x = \frac{Ae^{Bx}}{1 + \frac{AG}{B} * e^{Bx} - 1}$
19	1979	Siler	$\mu_x = A * exp^{-Bx} + C + D * exp^{E_x}$
20	1980	Heligman-Pollard	$q_x/p_x = A^{(x+B)^C} + D * exp^{-E*log\left(\frac{x}{F}\right)^2} + G * H^x$
21	1980	Heligman-Pollard	$q_x = A^{(x+B)^C} + D * exp^{-E*log\left(\frac{x}{F}\right)^2} + \frac{G*H^x}{1+G*H^x}$
22	1980	Heligman-Pollard	$q_x = A^{(x+B)^C} + D * exp^{-E*log\left(\frac{x}{F}\right)^2} + \frac{G*H^x}{1+K*G*H^x}$
23	1980	Heligman-Pollard	$q_x = A^{(x+B)^C} + D * exp^{-E*log\left(\frac{x}{F}\right)^2} + \frac{G*H^{x^k}}{1+K*G*H^{x^k}}$

24	1983	Rogers-Planck	$q_x = A_0 + A_1 e^{-Ax} + A_2 e^{B(x-u)-e^{-C(x-u)}} + A_3 e^{Dx}$
25	1987	Martinelle	$\mu_x = \frac{Ae^{BX}+C}{1+De^{BX}} + Ke^{BX}$
26	1992	Carriere	$l_x = P_1 l(x)(Weibul) + P_2 l(x)(Inweibul) + P_3 l(x)(gompertz)$
27	1992	Carriere	$l_x = P_1 l(x)(Weibul) + P_2 l(x)(Ingompertz) + P_3 l(x)(gompertz)$
28	1992	Kostaki	$\frac{q_x}{p_x} = A^{(x+b)^C} + \mathrm{D}\ e^{E_i log(\frac{x}{f})^2} + GH^x$
29	1998	Kannisto	$mu[x] = \frac{Ae^{Bx}}{1+= Ae^{Bx}})$
30	1998	Kannisto-Makeham	$mu[x] = \frac{Ae^{Bx}}{1+= Ae^{Bx}}) + C$

Models given in table 3.3 are fitted to the data and parameters of the models are estimated. The loss functions of each models are calculated using three different loss functions given in table 2. The models which produce lowest loss function and provides more reliable results under the three loss functions are used for graduating mortality. Assumptions of loss function/residual are tested to confirm their independence and the normality assumptions. For this purpose, R Software is used. The details of the parametric mortality models are provided in table 3.2.The parametric models are applied towards the mortality pattern of the urban and rural mortality of Pakistan. The Square root error of all the models given in the table 3 are determined using the three loss functions given in table 2. we calculate reliability of square root error of each model under three loss functions and rank models based on highest reliable square root errors under three loss functions. The root square errors of top seven models under three loss functions are given in table 3.

Table 4

Residual Standard Errors and Reliability of top Seven Models

S.No	Model	RSE- Loss Funtion-1	RSE- Loss Function-2	RSE-Loss Function-3	Reliability of the output/S.E
		Urban Areas			
1	Rogers-Planck	0.03969	0.04008	0.04014	0.00024
2	Carriere	0.04455	0.04101	0.04318	0.00179
3	Carriere2	0.04794	0.04248	0.03992	0.00410
4	Heligman-Pollard	0.06206	0.04269	0.04638	0.01028
5	Heligman-Pollard2	0.06160	0.04269	0.04638	0.01002
6	Kostaki	0.06248	0.04309	0.04639	0.01037
7	Siler	0.07469	0.04734	0.04345	0.01702
		Rural Areas			
1	Siler	0.09130	0.04148	0.07454	0.02535
2	Wittstein	0.06350	0.05798	0.04392	0.01010
3	Heligman-Pollard	0.05006	0.04216	0.04390	0.00415
4	Rogers-Planck	0.04732	0.03835	0.03781	0.00534
5	Heligman-Pollard2	0.05008	0.04289	0.04390	0.00389
6	Heligman-Pollard3	0.04560	0.03869	0.04058	0.00357
7	Heligman-Pollard4	0.04961	0.03995	0.03506	0.00740

We examine the assumptions of residual of top seven models by conducting graphical residual analysis and found that following models with the given loss function used for modelling the urban mortality, validate residual assumptions:

1. Carriere Model with Loss Function 2
2. Carriere Model with Loss Function 3
3. Helligman Polard's Model 1 with Loss Function 3
4. Helligman Polard's Model 2 with Loss Function 3
5. Kostaki's Model with Loss Function 3

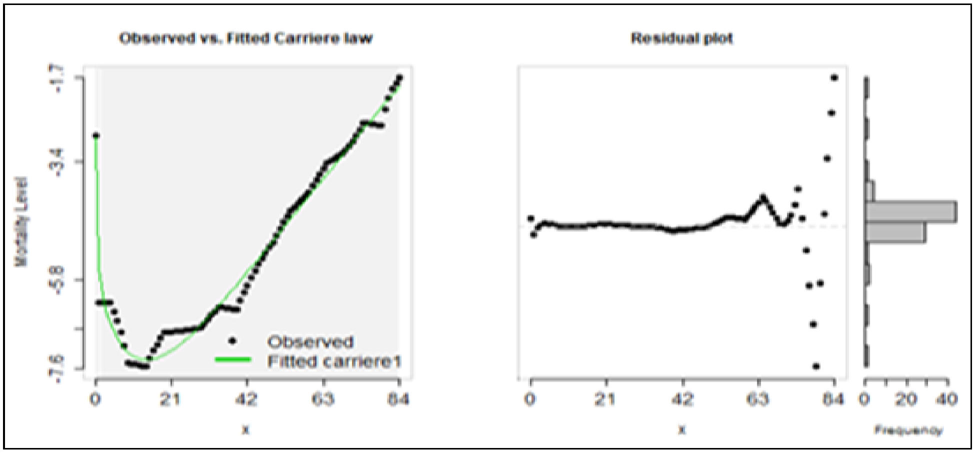
Similarly, we also examine residuals of models used for modelling rural mortality and found that the following models confirm the residuals assumptions:

1. Helligman Polard's Model 1 with Loss Function 3
2. Rogerds-Planck Model with Loss Function 3
3. Helligman Polard's Model 2 with Loss Function 3
4. Helligman Polard's Model 3 with Loss Function 3
5. Helligman Polard's Model 4 with Loss Function 3

Next we chose one model each for graduating the urban and rural mortality based on the lowest square root error. It is observed that Carriere 2 Model with Loss Function 3 provides the lowest residual for modelling urban mortality, whereas Heligman Ploards' model with loss function 3 provides lowest standard error for modelling the rural population. In this connection, residual analysis of the Carriere model-2 is conducted. The histogram of the residuals, and residuals plot as well as the observed and fitted values of the mortality rates are of urban areas is given in Figure 1.

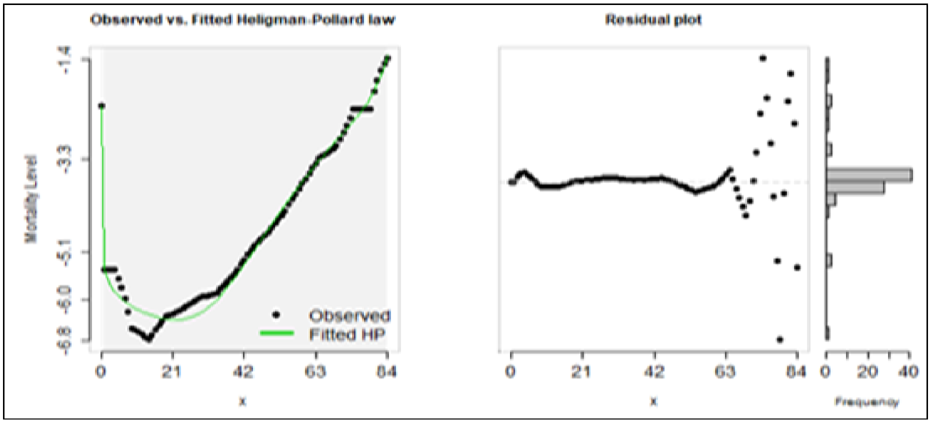
Figure 2

Observed Vs. Fitted Values, Residuals Plot & Residuals Frequency Distribution of Carriere Model



The histogram of the residuals as well as residual plots of Carriere model-2 indicates that the residuals are random and are approximately normally distributed. Furthermore, the observed and fitted values of mortality rates graph indicates that the proposed model perfectly estimates mortality rates of urban areas. Residual analysis of Heligman-Polard's model-4 is conducted. The histogram of the residuals, and residuals plot as well as the observed and fitted values of the mortality rates are of urban areas is given in Figure 2. The histogram of the residuals as well as the residual plots of Heligman-Polard's model-4 indicates that the residuals are random and are approximately normally distributed. Furthermore, observed and fitted values of mortality rates graph indicates that the proposed model perfectly estimates the mortality rates of rural areas.

Figure 2
Observed Vs. Fitted, Residuals Plot & Residuals Frequency Distribution Heligman-Polard's Model



RESULTS OF STUDY

The proposed models are used to determine mortality rates of urban and rural areas of Pakistan. Based on Carriere model, the mortality rates of urban areas at each age up to maximum age 85 are calculated and presented in table 5.

Table 5
Graduated Mortality Rates of Urban Areas

Age	Mortality Rates	Age	Mortality Rates	Age	Mortality Rates
0	0.054704284	29	0.001194596	58	0.014996348
1	0.003804334	30	0.001288883	59	0.016428069
2	0.002255077	31	0.001393176	60	0.017996708
3	0.001620971	32	0.001508368	61	0.019715153
4	0.001277855	33	0.001635448	62	0.021597457
5	0.001065433	34	0.001775503	63	0.023658937
6	0.000923312	35	0.001929732	64	0.025916264
7	0.000823630	36	0.002099455	65	0.028387560
8	0.000751774	37	0.002286122	66	0.031092497
9	0.000699371	38	0.002491326	67	0.034052387
10	0.000661309	39	0.002716816	68	0.037290265
11	0.000634318	40	0.002964515	69	0.040830955

12	0.000616234	41	0.003236529	70	0.044701101
13	0.000605597	42	0.003535170	71	0.048929172
14	0.000601414	43	0.003862974	72	0.053545380
15	0.000603011	44	0.004222721	73	0.058581517
16	0.000609944	45	0.004617459	74	0.064070644
17	0.000621943	46	0.005050529	75	0.070046568
18	0.000638868	47	0.005525593	76	0.076543018
19	0.000660687	48	0.006046661	77	0.083592372
20	0.000687455	49	0.006618130	78	0.091223742
21	0.000719306	50	0.007244813	79	0.099460103
22	0.000756442	51	0.007931983	80	0.108314046
23	0.000799130	52	0.008685413	81	0.117781513
24	0.000847697	53	0.009511422	82	0.127832609
25	0.000902535	54	0.010416929	83	0.138398225
26	0.000964094	55	0.011409501	84	0.149350757
27	0.001032891	56	0.012497419	85	0.160476839
28	0.001109507	57	0.013689736		

Similarly, the mortality rates of rural areas are calculated for the entire ages range using the Heligman-Polard's model-4. The mortality rates of rural areas are given in table 5.

Table 6
Graduated Mortality Rates of Rural Areas

Age	Mortality Rates	Age	Mortality Rates	Age	Mortality Rates
0	0.102008896	29	0.001891413	58	0.021544575
1	0.004473152	30	0.001970380	59	0.023487445
2	0.003594221	31	0.002065624	60	0.025574413
3	0.003149278	32	0.002178935	61	0.027812789
4	0.002861914	33	0.002312257	62	0.030210119
5	0.002654255	34	0.002467695	63	0.032774260
6	0.002494031	35	0.002647516	64	0.035513487
7	0.002364976	36	0.002854161	65	0.038436690
8	0.002257843	37	0.003090241	66	0.041553677
9	0.002166928	38	0.003358552	67	0.044875671
10	0.002088510	39	0.003662073	68	0.048416077
11	0.002020066	40	0.004003972	69	0.052191697
12	0.001959840	41	0.004387612	70	0.056224620
13	0.001906606	42	0.004816554	71	0.060545142
14	0.001859507	43	0.005294562	72	0.065196281
15	0.001817973	44	0.005825601	73	0.070240721
16	0.001781652	45	0.006413847	74	0.075771410
17	0.001750376	46	0.007063682	75	0.081927665
18	0.001724133	47	0.007779701	76	0.088919371
19	0.001703049	48	0.008566705	77	0.097062792
20	0.001687379	49	0.009429709	78	0.106832328

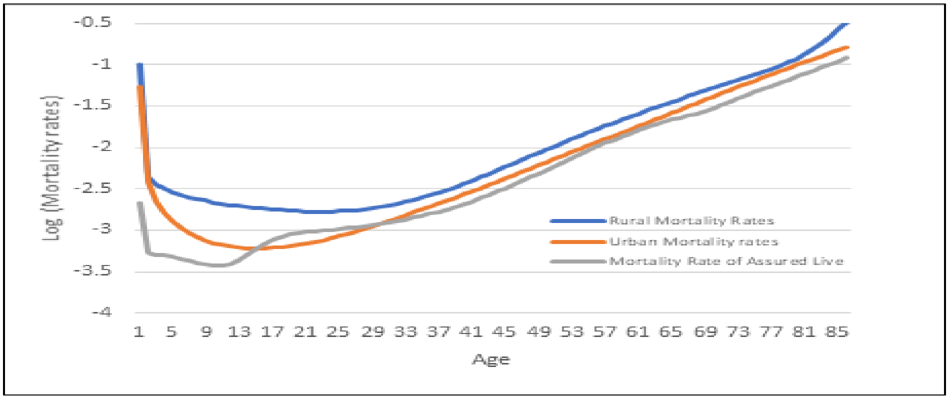
21	0.001677502	50	0.010373935	79	0.118932382
22	0.001673912	51	0.011404810	80	0.134390506
23	0.001677223	52	0.012527967	81	0.154662804
24	0.001688164	53	0.013749235	82	0.181717955
25	0.001707586	54	0.015074638	83	0.218018561
26	0.001736459	55	0.016510389	84	0.266250527
27	0.001775882	56	0.018062884	85	0.328611488
28	0.001827079	57	0.019738696		

DISCUSSION

The graduated mortality rates of urban and rural areas are depicted in Fig 5.1 which shows that the mortality rates of the urban areas are lower than the rural areas. The higher mortality rates in urban areas could be contributed to the several reasons including but not limited to the non-availability of the medical facilities in the rural areas, the level of education, the awareness about health and medical condition, life style, economic condition of the people living in rural areas, mindset of people about the medical facility, social and cultural values, and many more factors.

Figure 4

Comparison of Mortality Rates



We have also compared the general graduated mortality rates with the graduated mortality rates of assured lives. The assured lives' mortality is significantly lower than the graduated mortality rates of the rural areas whereas the graduated mortality of urban areas is closer to the graduated mortality of insured lived over the age range 29-65, this could be possibly due the reasons that majority of the insured lives belong to urban areas, however, significant variation exist at age range 0 - 17 mortality rate of assured lives is significantly lower than the urban population. The graduated mortality at ages ≥ 65 are lower than the both the urban and rural population.

CONCLUSION

In this study we have examined around thirty parameteric mortality models for graduating the mortality rates of urban and rural population of Pakistan. We have used the root square error and variability in the root square error under various loss functions to compare these models. Our analysis indicates that the Heligman-Polards model- 4 with loss function -3 has the more stable and lowest Soot Square Errors lower variation across all the loss functions used in the study. Based on our analysis, we suggest that Heligman-Polards's model-4 with loss function 3 should be used graduating mortality of the rural population of Pakistan. Similarly, we propose Cariere's Model-2 with loss function 3 for graduating mortality of urban areas of Pakistan at all ages. Using the two parametric mortality models, we have graduated the mortality rates of urban and rural populations of Pakistan at each age over the age range [0,85]. Therefore, the graduated mortality rates are quite different from each other. Furthermore, the mortality rates of the urban and the rural areas can be used of these mortality rates for pricing the insurance products.

Future Research

This work graduates the mortality rates at each age of urban and rural population in Pakistan that would be helpful for life insurance companies when they price their products based on historic mortality pattern, however, it would be more useful to forecast the mortality rates in future to provide the greater insight in the prospective mortality rates for policy makers and insurance companies to adjust their policies and premium pricing as well as liabilities reserves respectively, in which case this work could be extended by employing the stochastic mortality models. There are significant differences between the graduated mortality rates of urban and rural areas, these differences need to be explored further to determine causes of this variation. Another area that need to be explored further is about the difference between mortality rates assure lives and rural mortality rates to explore that whether these variations are caused by the underwriting activity/practice or the mortality rates of the assure lives are driven by the urban population.

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