NUMERICAL METHOD OF MODIFIED NEWTON RAPHSON METHOD WITHOUT SECOND DERIVATIVE FOR SOLVING THE NONLINEAR EQUATIONS

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KEYWORDS	ABSTRACT
Nonlinear Equations, Second Order Methods, Difference Operator, Numerical Analysis.	This paper presents a Numerical Method of Modified Newton Raphson method for Solving the Nonlinear equations. Finding the root of nonlinear equations is one of classical problem in numerical analysis, which arises frequently in various branches of science and engineering. The aim of this paper is to develop Numerical Method to find the approximation of the root of nonlinear equation without estimation second order derivatives. The proposed Method depend on Modified Newton Raphson method and difference operator. The benefit of proposed method is that it is converged quadratically, and it doesn't require to compute second derivative. The proposed method is very efficient and convenient for solving root of nonlinear equations. Its theoretical results and high computational efficiency is confirmed by Numerical examples.

INTRODUCTION

The determination of nonlinear equations is much significant in applied science and especially in engineering, and techniques which we can solve of these nonlinear equations by using Numerical Methods. Development of numerical methods for solving nonlinear equations just because analytic methods fail to solve the polynomial equations of higher degree and the transcendental equations. Due to this purpose, several basic numerical root location methods have endorsement for solving nonlinear equations. The methods are Bisection method, Regula Falsi method, Secant method, fixed point method, Newton Raphson method etc. These methods are used to solve for many complicated problems including applied science and engineering. These methods have been discussed in several literatures comprehensively (Solanki, 2014; Allame & Azad, 2012; Ali, 2015; Liu & Zhang, 2014; Qureshi, Ansari & Syed, 2018; Masoud & Azad, 2012; Qureshi, kalhoro Bhutto, Khokar & Qureshi, 2018).

Furthermore, piles of numerical methods had been developed, which is free from 1st and 2nd derivatives that are simpler, easier to use and free from any pitfall by (Qureshi et al, 2018), (Soomro, 2016; Qureshi, Shaikh & Solanki, 2017; Liu et al, 2014; Qureshi, Shaikh & Malhi, 2018; Siyal, 2016; Ranbir, Roy, Yaikhom & Takhellambam, 2013; Saba & Ann, 2015; Sangah, 2016). In light of above research, this paper has been developed numerical method for estimating a single root of nonlinear problems. The purpose of new iterated method is proposing a mathematical tool for solving all possible root of polynomial of higher degree functions and transcendental functions. C++/MATLAB is used to defend the numerical results and it is compared with Newton Raphson method and Modified Newton Raphson method. The proposed iterated method has rapidly converged and free from pitfall as assessment of existing second convergence numerical method for solving nonlinear equations.

NUMERICAL AGORITHM

This segment has developed a numerical method for solving nonlinear equations. The developed modification has been done by using Modified Newton Raphson Method and difference operator, such as Modified Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{[f'(x_n)]^2 - f(x_n)\, f''(x_n)} \quad ---------- (1)$$

Free it second derivative by using difference operator, such as

$$f''(x_n) = \frac{f`(x_n + h) - f'(x_n)}{h} \quad --------- (2)$$

Some what `h` can be written as

$$h = \Delta x_n = f\,(x_n) \quad -------- (3)$$

Using Eq. (3) in Eq. (2), we get

$$f''(x_n) = \frac{f`\left(x_n + f(x_n)\right) - f'(x_n)}{f(x_n)} \quad --------- (4)$$

Now, using Eq. (4) in Eq. (1), we get

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{[f'(x_n)]^2 - f`\left(x_n + f(x_n)\right) + f'(x_n)}$$

Finally, we get

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{f'(x_n)[f'(x_n) + 1] - f`(x_n + f(x_n))} \quad ------- (5)$$

Hence, Eq. (5)is a Numerical Method of Modified Newton Raphson Method without Second Derivative.

NUMERICAL CONVERGENCE

The following section will be illustration that the Developed Numerical Method is Converge Quadratically.

Theorem: Let a ∈ I be a simple zero of sufficiently differentiable function f: I ⊆ R → R for an open interval I. If x_0 is sufficiently close to `a`, then the proposed numerical iterative method has 2nd-order convergence.

Proof:

Using the relation $e_n = x_n - a$ in Taylor series, we are estimating $f(x_n), f`(x_n)$ and $f`\left(x_n + f(x_n)\right)$ with using this condition c = $\frac{f``(a)}{2f`(a)}$ and ignoring higher order term, such as

$$f(x_n) = f`(a)(e_n + c{e_n}^2) \quad --------------- (i)$$

$$f`(x_n) = f`(a)(1 + 2ce_n) \quad --------------- (ii)$$

and

$$f\left(x_n + f(x_n)\right) = f`(a)[\left(e_n + f(x_n)\right) + c\left(e_n + f(x_n)\right)^2 \quad ----- (iii)$$

or

$$f`\left(x_n + f(x_n)\right) = f`(a)[1 + f`(x_n)]\left[1 + 2c\left(e_n + f(x_n)\right)\right] \quad ---- (iv)$$

By using Eq. (i)and Eq. (ii) in Eq. (iii),

$$f`\left(x_n + f(x_n)\right) = f`(a)\,[1 + f`(a) + 2ce_n[1 + 3f`(a) + f`^2(a)]] -- (v)$$

Subtracting $Eq.$ (iv) by $Eq.$ (ii), we get

$$f`(x_n + f(x_n)) - f`(x_n) = f`(a)\ [f`(a) + 2ce_n[3f`(a) + f`^2(a)]] \quad --- (v)$$

By using $Eq.(i),\ Eq.(ii) and\ Eq.(v)$ in developed method, we get

$$e_{n+1} = e_n - \frac{f`(a)e_n(1\ + ce_n)f`(a)(1 + 2ce_n)}{f`(a)\{(1 + 2ce_n)(1 + 2ce_n) -\ [f`(a) + 2ce_n[3f`(a) + f`^2(a)]]\}}$$

$$e_{n+1} = e_n - \frac{f`(a)e_n(1 + 3ce_n)}{1 -\ f`(a) - 2ce_n[-2 + 3f`(a) + f`^2(a)]]}$$

$$e_{n+1} = e_n - f`(a)e_n(1 + 3ce_n)[1 -\ f`(a) - 2ce_n[-2 + 3f`(a) + f`^2(a)]]^{-1}$$

$$e_{n+1} = e_n - e_nf`(a)(1 + 3ce_n)[1 + f`(a) + 2ce_n[-2 + 3f`(a) + f`^2(a)]]$$

$$e_{n+1} = [1 - f`(a) - f`^2(a)]e_n + 2ce^2{}_nf`(a)[2 - 3f`(a) - f`^2(a) + 3cf`(a) + 3cf`^2(a)] \quad --- (vi)$$

for nonlinear equation such as $f(x_n) = 0$ in using Eq. (i) then substitute in Eq. (vi), we get

$$e_{n+1} = [1 - f`(a) - f`^2(a)]ce_n{}^2 + 2ce^2{}_nf`(a)[2 - 3f`(a) - f`^2(a) + 3cf`(a) + 3cf`^2(a)]$$

$$e_{n+1} = e^2{}_n[c + 3cf`(a) - c(7 - 6c)f`^2(a) - c(2 - 6c)f`^3(a)] \qquad --- (vii)$$

Hence, this is proven that the proposed numerical iterative method is Converge Quadratic.

NUMERICAL RESULTS

This segment few of numerical examples have employed by developed method for solving nonlinear problems and compare with Newton Raphson Method and Modified Newton Raphson Method (Table-1). Numerical calculations have been done by C++/MATLAB with accuracy ∈< 10. From numerical results it has been observed that the developed numerical method is better than existing second order methods, such as in Table-1

Table 1 Numerical Calculations

FUNCTIONS	METHODS	ITERATIONS	ROOT	AE%
$f(x) = x^2 - e^x - 3x + 2$	N R Method	5	0.25753	2.98023e-008
	M N R Method	6	0.25753	5.96046e-008
	New Method	4	0.25753	9.53674e-007
$f(x) = e^x - 4x$	N R Method	5	1.93456	1.19209e-007
	M N R Method	4	1.93456	1.67012e-004
	New Method	4	1.93456	1.69277e-005
$f(x) = sinx - x - 1$	N R Method	4	0.35743	5.96046e-008
	M N R Method	3	0.35743	1.53184e-005
	New Method	3	0.35743	1.13428e-005
$f(x) = x^2 - 2x - 5$	N R Method	5	1.33456	2.98023e-008
	M N R Method	4	1.33456	1.78814e-007
	New Method	3	1.33456	2.00021e-003
$f(x) = 2x - \ln x - 7$	N R Method	4	4.21996	4.76837e-007
	M N R Method	3	4.21996	4.76837e-006
	New Method	3	4.21996	1.21117e-006
$f(x) = e^x - 5x$	N R Method	4	0.25743	5.96046e-008
	M N R Method	3	0.25743	1.53184e-005
	New Method	3	0.25743	7.89762e-006

CONCLUSION

In this research, we have suggested a numerical method for solving nonlinear equations. This method is based on the difference operator and Modified Newton Raphson Method. By theoretical analysis and numerical experiments, we confirm that the proposed second order numerical method has high computational efficiency as the assessment of existing methods. We can see that suggested method is suitable for solving nonlinear equations. Finally, it has been observed from numerical results that the proposed numerical method of modified newton raphson method has kept good efficiency and performing supercilious for solving nonlinear equations.

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